✶ PROGRAMA DE VERÃO 2012 ✶

# SISTEMAS DINÂMICOS

## Floer homology and applications

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In lecture 1 we discuss the finite dimensional toy model for Floer homology, the Morse-Witten complex associated to a closed Riemannian manifold *M* equipped with a Morse function. We present its construction via ODE and dynamical systems methods. The Morse-Witten complex greatly illustrates the general case and provides many useful pictures to keep in mind. It is naturally isomorphic to singular homology of *M*. In lecture 2 we introduce Floer's elliptic PDE and define Floer homology for (aspherical) closed symplectic manifolds *N*. It is a symplectic invariant. We show that it is naturally isomorphic to singular homology of *N*. Thereby we recover Floer's proof of the Arnold conjecture. Throughout lectures 3 and 4 we generalize the construction to symplectic manifolds N with boundary of contact type. In order to still get a symplectic invariant in the presence of a boundary we need to put Floer homology into the algebraic framework of directed systems. Symplectic homology is defined by taking the direct limit. We will calculate it for the unit ball in $R^{2n}$ with the standard symplectic structure $\omega_0$. The outcome of this calculation is used to prove the Weinstein conjecture for closed hypersurfaces of contact type in $(R^{2n}; \omega_0)$. In lectures 5 and 6 we introduce Floer homology for a class of noncompact, but exact (hence aspherical), symplectic manifolds, namely cotangent bundles *T*M*. It is naturally isomorphic to singular homology of the free loop space of *M*. We indicate three methods of this calculation and present some details of the heat flow method. Here the main analytical difficulty is to deform Floer's elliptic PDE on *T*M* into the parabolic heat equation on the closed Riemannian manifold *M* keeping track of the solutions. Now we meet the Morse-Witten complex of lecture 1 again, but this time the manifold is the infinite dimensional free loop space *M* and the Morse function is the classical action functional whose critical points are (perturbed) closed geodesics in *M*. Flow lines are provided by the solutions to the heat equation. At this point we have identified Floer homology with this version of Morse homology. To actually calculate the latter we profit from the fact that many tools from finite dimensional dynamical systems are still available for parabolic PDE's. On the other hand, having only a semiflow on *M* requires new methods to construct a suitable Morse filtration (the key object to obtain a natural isomorphism to singular homology). Here Conley index theory enters.

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